RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, JUNE 2022
FIRST YEAR (BATCH 2021-24)
MATHEMATICS (GENERAL)
Paper : II

Date : 24/06/2022
Time : 11.00 am – 2.00 pm
Full Marks : 75

[Use a separate Answer book for **each group**]

Group – A

Answer **any five** questions of the following: [5×5]

1. Solve: $\left(D^2+4\right)y = x\sin^2 x$, where $D \equiv \frac{d}{dx}$. (5)

2. Solve: $y = px + \frac{a}{p}, p = \frac{dy}{dx}$. (5)

3. Consider $\left(D^2-2D\right)y = e^x \cos x, D \equiv \frac{d}{dx}$ and solve it by the method of variation of parameters. (5)

4. Solve: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. (5)

5. Find $\frac{1}{D^3-5D^2+8D-4}e^{2x}$,where $D \equiv \frac{d}{dx}$. (5)

6. Solve **not** by using the method of variation of parameters : $y_2 + a^2 y = \sec ax.$ (5)

7. a) Solve: $\left(x^3+xy^4\right)dx + 2y^3 dy = 0.$ (3)

   b) Solve: $xdy - ydx = \left(\cos\frac{1}{x}\right)dx.$ (2)

8. Reduce $x^2p^2 + y(2x+y)p + y^2 = 0$ to Clairaut's form by the substitution $y = u, xy = v.$ Hence solve the equation, $p = \frac{dy}{dx}$. (5)

Group – B

Answer **any five** questions of the following: [5×10]

9. a) State Leibnitz's Theorem on successive differentiation.

   b) If $y = (\sin^{-1} x)^2$, prove that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

   c) Prove that $\lim_{x\to 0} \log_{\tan^2 x}\left(\tan^2 2x\right) = 1.$ (2+5+3)

10. a) Find the asymptotes of the curve:
    $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.

    b) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are connected by the relation $a^2b^3 = c^5$. (5+5)

11. a) Evaluate $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}}$, by changing the order of integration.

b) Find the values of a and b so that $\lim_{x\to 0} \frac{a\sin 2x - b\sin x}{x^3} = 1$ (6+4)

12. a) Find the volume of the solid obtained by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about its axis of symmetry.

b) Find the surface formed by revolution of $x^2 + 4y^2 = 16$ about x - axis. (5+5)

13. a) Using Beta function, show that $\int_0^2 x(8-x^3)\,dx = \frac{16\pi}{9\sqrt{3}}$

b) Show that $\int_0^1 x^p (1-x^q)^n\,dx = \frac{1}{q}\beta\left(\frac{p+1}{q}, n+1\right);\ p,q,n>0$. (5+5)

14. a) Show that repeated limits of the function

$$f(x,y) = \begin{cases} \dfrac{xy}{xy+x-y} & ; \quad (x,y)\neq(0,0) \\ o & ; \quad (x,y)=(0,0) \end{cases}$$

exist and are equal but double limit does not exist as $(x,y)\to(0,0)$.

b) Using the definition, prove that $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2} = 0$. (6+4)

15. a) If $f(x,y)$ be continuous at $(0,0)$, then prove that the functions $f(x,b)$ and $f(a,y)$ are continuous at $x=a$ and $y=b$ respectively.

b) Give an example, where the converse of the above result is not true. (6+4)

16. a) Prove that between any two real roots of $e^x \sin x = 1$, there exists at least one real root of $e^x \cos x + 1 = 0$.

b) Prove that between two real roots of the equation $e^x \sin x + 1 = 0$, there is at least one real root of $\tan x + 1 = 0$. (4+6)

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